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DARK MATTER

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Abstract. I give a review of the development of the concept of dark matter. The dark matter story passed through several stages from a minor observational puzzle to a major challenge for theory of elementary particles. Modern data suggest that dark matter is the dominant matter component in the Universe, and that it consists of some unknown non-baryonic particles. Properties of dark matter particles determine the structure of the cosmic web.

Key words: Dark matter, galaxies, clusters of galaxies, large-scale structure of the Universe

1. INTRODUCTION

The possible presence of invisible but gravitating dark matter has been studied already almost hundred years. First attempts to derive the total density of matter in the Solar vicinity were made by Öpik (1915), Kapteyn (1922), Jeans (1922), and Oort (1932). Results were different: Öpik (1915), Kapteyn (1922) and Oort (1932) found that the total density of matter can be explained by known stellar populations, if a reasonable extrapolation of faint dwarf stars is taken into account. In contrast, Jeans (1922) found that there must be two dark stars to each bright star. This discussion continued until the end of 20th century.

A much larger discrepancy between the masses of visible objects and the total masses of stellar systems they belong to, was discovered by Zwicky (1933). He concluded that, in order to hold galaxies together in the cluster, the cluster must contain huge amounts of dark (invisible) matter.

Initially no distinction between local dark matter in the solar vicinity and global dark matter in clusters of galaxies was made. The realisation, that these two types of dark matter have very different properties and nature came from the detailed study of galactic models (Einasto, 1974).

In this talk I have used my recent review of the dark matter (Einasto, 2009).

2. LOCAL DARK MATTER

The dynamical density of matter in the solar vicinity can be estimated using vertical oscillations of stars around the galactic plane. Öpik (1915) found that

the summed contribution of all known stellar populations and interstellar gas is sufficient to explain the vertical oscillations of stars – in other words, there is no need to assume the existence of a dark population. Similar analyses were made by Kapteyn (1922) and Jeans (1922), who used the term "Dark Matter" to denote the invisible matter. Kapteyn found for the dynamical density of matter near the Sun 0.099 $M_\odot/pc^3$, Jeans got 0.143 in the same units.

Oort (1932) analysis suggested that the total density is about 0.092 $M_\odot/pc^3$, and the density of stars, including expected number of white dwarfs, is approximately equal to the dynamical density. He concluded that the total mass of nebulous or meteoric dark matter near the Sun is very small.

Kuzmin (1952, 1955) and his students Eelsalu (1959) and Jõeveer (1972, 1974) confirmed the earlier results by Öpik, Kapteyn and Oort. A number of other astronomers, including more recently Oort (1960) and Bahcall (1984, 1987), found results in agreement with the Jeans result. Their results mean that the amount of invisible matter in the solar vicinity should be approximately equal to a half of the amount of visible matter.

Modern data by Kuijken & Gilmore (1989); Gilmore et al. (1989) have confirmed the results by Kapteyn (1922), Oort (1932), Kuzmin (1952, 1955) and his collaborators. Thus we come to the conclusion that *there is no evidence for the presence of large amounts of dark matter in the disk of the Galaxy.* If there is some invisible matter near the galactic plane, then its amount is small, of the order of 15 percent of the total mass density.

3. GLOBAL DARK MATTER

Zwicky (1933, 1937) measured radial velocities of galaxies in the Coma cluster of galaxies, and calculated the mean random velocities in respect to the mean velocity of the cluster. Galaxies move in clusters along their orbits; the orbital velocities are balanced by the total gravity of the cluster. Zwicky found that orbital velocities are almost a factor of ten larger than expected from the summed mass of all galaxies belonging to the cluster.

A certain discrepancy was detected between masses of individual galaxies and masses of pairs and groups of galaxies (Holmberg, 1937; Page, 1952, 1959, 1960). These determinations yield for the mass-to-light ratio (in blue light) the values $M/L_B = 1 \ldots 20$ for spiral galaxy dominated pairs, and $M/L_B = 5 \ldots 90$ for elliptical galaxy dominated pairs. These ratios are larger than found from local mass indicators of galaxies (velocity dispersions at the centre and rotation curves of spiral galaxies).

Kahn & Woltjer (1959) paid attention to the fact that most galaxies have positive redshifts, only the Andromeda galaxy (M31) has a negative redshift of about 120 km/s, directed toward our Galaxy. This fact can be explained, if both galaxies form a physical system. From the approaching velocity, the mutual distance, and the time since passing the perigalacticon (taken equal to the present age of the Universe), the authors calculated the total mass of the double system. They found that $M_{tot} \geq 1.8 \times 10^{12}$ $M_\odot$. The conventional masses of the Galaxy and M31 were estimated to be of the order of 2×10^{11} $M_\odot$. In other words, the authors found evidence for the presence of additional mass in the Local Group of galaxies.

Information of masses of individual galaxies come from their rotation velocities. Roberts (1966) made a 21-cm hydrogen line survey of M31 using the National

Radio Astronomy Observatory large 300-foot telescope. He found that the rotation velocity curve at large radii is flat, i.e., velocity is almost constant. From the comparison of the light distribution with the rotation curve the local value of the mass-to-luminosity can be calculated. He found in the outer regions a mass-to-light ratio ~ 250. A similar high value was found for the edge-on S0 galaxy NGC 3115 by Oort (1940). Rubin & Ford (1970) and Roberts & Rots (1973) derived the rotation curve of M31 up to a distance ~ 30 kpc, using optical and radio data, respectively. The rotation speed rises slowly with increasing distance from the centre of the galaxy and remains almost constant over radial distances of 16–30 kpc.

Two possibilities were suggested to explain flat rotation curves of galaxies. One possibility is to identify the observed rotation velocity with the circular velocity. In this case an explanation for a very high local M/L should be found. To explain this phenomenon it was suggested that in outer regions of galaxies low-mass dwarf stars dominate (Oort, 1940; Roberts, 1975). The other possibility is to assume that in the periphery of galaxies there exist non-circular motions which distort the rotation velocity.

4. GALACTIC MODELS

Classical models of elliptical galaxies were found from luminosity profiles and were calibrated using either central velocity dispersions, or motions of companion galaxies. Models of spiral galaxies were constructed using rotation velocities. A natural generalisation of classical galactic models is the use of all available observational data – photometric data on the distribution of colour and light, and kinematical data on the rotation and/or velocity dispersion. Further, it is natural to include into models data of all major stellar populations, such as the bulge, the disk, the halo, as well as the flat population in spiral galaxies, consisting of young stars and interstellar gas.

All principal descriptive functions of galaxies (circular velocity, gravitational potential, projected density) are simple integrals of the spatial density. Therefore it is natural to apply for the spatial density $\rho(a)$ of galactic populations a simple generalised exponential expression (Einasto, 1965):

$$\rho(a) = \rho(0) \exp\left(-(a/a_0)^{1/N}\right), \tag{1}$$

where a is the semi-major axis of the isodensity ellipsoid, a_0 is the effective radius of the population, and N is a structural parameter, determining the shape of the density profile. This expression (called the Einasto profile) can be used for all galactic populations, including dark halos. The case $N = 4$ corresponds to the de Vaucouleurs density law for spheroidal populations, $N = 1$ corresponds to the exponential density law for disk.

To combine photometric and kinematic data, mass-to-light ratios of galactic populations are needed. Luminosities and colours of galaxies in various photometric systems result from the physical evolution of stellar populations that can be modelled. Detailed models of the physical and chemical evolution of galaxies were constructed by Tinsley (1968). Combined population and physical evolution models were calculated for a representative sample of galaxies by Einasto (1972). It is natural to expect, that in similar physical conditions the mass-to-luminosity ratio M_i/L_i of the population i has similar values in different stellar systems (star

clusters, galactic populations). Thus star clusters and central cores of galaxies can be used to estimate M_i/L_i values for the main galactic populations.

5. GALACTIC CORONAS

Results of these calculations were reported at the First European Astronomy Meeting by Einasto (1974). The main conclusion was: it is impossible to reproduce the rotation data by known stellar populations only. The only way to eliminate the conflict between photometric and rotational data was *to assume the presence of an unknown almost spherical population with a very high value of the mass-to-light ratio, large radius and mass.* To avoid confusion with the conventional stellar halo, the term "corona" was suggested for the massive population. Thus, the detailed modelling confirmed earlier results obtained by simpler models. But here we have one serious difficulty – no known stellar population has so large a M/L value.

Additional arguments for the presence of a spherical massive population in spiral galaxies came from the stability criteria against bar formation, suggested by Ostriker & Peebles (1973). Their numerical calculations demonstrated that flat systems become rapidly thicker and evolve to a bar-like body. In real spiral galaxies a thin population exists, and it has no bar-like form. To remain stable galaxies must have a massive spherical halo.

The rotation data available in the early 1970s allowed the determination the mass distribution in galaxies up to their visible edges. In order to find how large and massive galactic coronas or halos are, more distant test particles are needed. If halos are large enough, then in pairs of galaxies the companion galaxies are located inside the halo, and their relative velocities can be used instead of the galaxy rotation velocities to find the distribution of mass around giant galaxies. This test was made by Einasto et al. (1974). A similar study was made independently by Ostriker et al. (1974). Our results were first discussed in the Caucasus Winter School on Cosmology in January 1974 and in the Tallinn Conference on Dark Matter in January 1975 (Doroshkevich et al., 1975).

The mass of galactic coronas exceeds the mass of populations of known stars by one order of magnitude. According to new estimates the total mass density of matter in galaxies is 20% of the critical cosmological density. The data suggest that all giant galaxies have massive halos/coronas, thus dark matter must be the dynamically dominating population in the whole Universe.

Initially the presence of massive coronas/halos was met with scepticism. In the Third European Astronomical Meeting the principal discussion was between the supporters of the classical paradigm with conventional mass estimates of galaxies, and of the new one with dark matter. The major arguments supporting the classical paradigm were summarised by Materne & Tammann (1976). Their most serious argument was:

"Big Bang nucleosynthesis suggests a low-density Universe with the density parameter $\Omega \approx 0.05$*; the smoothness of the Hubble flow also favours a low-density Universe."*

Additional observational data gave strong support to the presence of massive coronas/halos. Available rotation data were summarised by Roberts (1975). Extended rotation curves were available for 14 galaxies. In all galaxies the local mass-to-light ratio in the periphery reached values over 100 in solar units. Rubin et al. (1978, 1980) measured optically the rotation curves of galaxies at

very large galactocentric distances. (Bosma, 1978) measured rotation data for 25 spiral galaxies with the Westerbork Synthesis Radio Telescope. Both results suggested that practically all spiral galaxies have extended flat rotation curves.

Another very important measurement was made by Faber & Jackson (1976); Faber et al. (1977); Faber & Gallagher (1979). They measured the central velocity dispersions for 25 elliptical galaxies and the rotation velocity of the Sombrero galaxy, just outside the main body of the bulge. Their data yielded for the bulge of the Sombrero galaxy a mass-to-light ratio $M/L = 3$, and for the mean mass-to-light ratios for elliptical galaxies about 7. These results showed that the mass-to-light ratios of stellar populations in spiral and elliptical galaxies are similar for a given colour, and the ratios are much lower than accepted in earlier studies based on the dynamics of groups and clusters. In other words, high mass-to-light ratios of groups and clusters of galaxies cannot be explained by visible galactic populations.

The distribution of the mass in clusters can be determined if the density and the temperature of the intra-cluster gas are known. These data can be measured by the Einstein X-ray orbiting observatory. The mass of Coma, Perseus and Virgo clusters was calculated from X-ray data by Bahcall & Sarazin (1977); Mathews (1978). The results confirmed previous estimates of masses made with the virial method using galaxies as test particles.

Finally, masses of clusters of galaxies can be measured using gravitational lensing of distant galaxies by clusters. The masses of clusters of galaxies determined using this method, confirm the results obtained by the virial theorem and the X-ray data (Fischer & Tyson, 1997; Fischer et al., 1997).

Earlier suggestions on the presence of mass discrepancy in galaxies and galaxy systems had been ignored by the astronomical community. This time new results were taken seriously. However, it was still not clear how to explain the controversy of the Big Bang nucleosynthesis and the smoothness of the Hubble flow, discussed by Materne & Tammann (1976).

6. THE NATURE OF DARK MATTER

The local dark matter is baryonic (low-mass stars or Jupiter-like objects), since non-baryonic matter is dissipationless and cannot form a highly flattened population.

The nature of the global dark matter has been a subject of discussion for long time. Initially it was suggested that in outer regions of galaxies low-mass dwarf stars dominate (Oort, 1940; Ostriker et al., 1974; Roberts, 1975). However, the stellar nature of galactic coronas/halos meets several difficulties. Coronas have larger dimensions than all known stellar populations, thus from hydrostatic equilibrium condition coronal stars must have much higher velocity dispersions than other populations. No fast-moving stars were found (Jaaniste & Saar, 1975). If the hypothetical population is of stellar origin, it must be formed much earlier than all known populations, because known stellar populations of different age and metallicity form a continuous sequence of kinematical and physical properties, and there is no place where to include this new population (Einasto, 1974). It is known that star formation is not an efficient process – usually in a contracting gas cloud only about 1 % of the mass is converted to stars. Thus we have a problem how to convert, in an early stage of the evolution of the Universe, a large fraction of

the primordial gas into this population of dark stars.

The nature of dark matter was the basic problem discussed in the Caucasus Winter School and in the Dark Matter Conference in Tallinn (Doroshkevich et al., 1975). Silk (1974); Komberg & Novikov (1975) showed that gaseous coronas of galaxies and clusters cannot consist of neutral gas since the intergalactic hot gas would ionise the coronal gas. A corona consisting of hot ionised gas would be observable. A fraction of the coronal matter around galaxies and in groups and clusters of galaxies consists indeed of the X-ray emitting hot gas, but the amount of this gas is not sufficient to explain the flat rotation curves of galaxies (Turner, 2003).

The baryonic nature (stars, gas) of the dark matter contradicts also the nucleosynthesis constraints mentioned already by Materne & Tammann (1976). A third very important observation was made which caused doubts to the baryonic matter as the dark matter candidate. The Cosmic Microwave Background radiation temperature and density fluctuations are much lower than the theoretically predicted limit 10^{-3} (see, for instance Parijskij (1978)).

Then astronomers considered the possible existence of non-baryonic particles, such as heavy neutrinos. This suggestion was made independently by several astronomers (Szalay & Marx (1976) and others). If dark matter consists of heavy neutrinos, then this helps to explain the paradox of small temperature fluctuations of the cosmic microwave background radiation. Dark matter starts to condense at early epoch and forms potential wells, after the recombination baryonic matter flows into these wells and forms galaxies. However, numerical simulations of the formation of the structure of the neutrino-dominated dark matter Universe demonstrated, that in this case only supercluster-scale systems can form (see below).

7. DARK MATTER AND LARGE-SCALE STRUCTURE OF THE UNIVERSE

After my talk at the Caucasus Winter School Zeldovich offered me collaboration in the study of the universe. He was developing a theory of formation of galaxies (the pancake theory, Zeldovich (1970)). A hierarchical clustering theory was suggested by Peebles (1971). Zeldovich asked for our help in solving the question: can we find some observational evidence which can be used to discriminate between these theories?

In solving the problem we used our previous experience in the study of galactic populations: kinematical and structural properties of populations hold the memory of their previous evolution. Random velocities of galaxies are of the order of several hundred km/s, thus during the whole lifetime of the Universe galaxies have moved from their place of origin only by about 1 h^{-1} Mpc (the Hubble constant is used in units of $H_0 = 100\ h$ km s^{-1} Mpc^{-1}). In other words – if there exist some regularities in the distribution of galaxies, these regularities must reflect the conditions in the Universe during the formation of galaxies.

Thus we had a leading idea how to solve the problem of galaxy formation: *We have to study the distribution of galaxies on larger scales.* The three-dimensional distribution of galaxies, groups and clusters of galaxies can be visualised using wedge-diagrams, invented just when we started our study. We prepared relatively thin wedge diagrams, and plotted in the same diagram galaxies, as well as groups and clusters of galaxies. In these diagrams regularity was clearly seen: *isolated*

galaxies and galaxy systems populated identical regions, and the space between these regions was empty. This picture was quite similar to the distribution of test particles in a numerical simulation of the evolution of the structure of the Universe.

We reported our results (Jõeveer & Einasto, 1978) at the IAU symposium on Large-Scale Structure of the Universe in Tallinn 1977, the first conference on this topic. The main results were:

1. galaxies, groups and clusters of galaxies are not randomly distributed but form chains, converging in superclusters;

2. the space between galaxy chains contains almost no galaxies and forms holes (voids) of diameter up to $\approx 70\ h^{-1}$ Mpc;

3. the whole picture of the distribution of galaxies and clusters resembles cells of a honeycomb, rather close to the picture predicted by Zeldovich (1978).

However, some important differences between the Zeldovich pancake model and observations were detected. First of all, there exists a rarefied population of test particles in voids absent in real data. This was the first indication for the presence of biasing in galaxy formation – there is primordial gas and dark matter in voids, but due to low-density no galaxy formation takes place here (Jõeveer et al., 1978; Einasto et al., 1980). The second difference lies in the structure of galaxy systems in high-density regions: in the model large-scale structures (superclusters) have rather diffuse forms, real superclusters consist of multiple intertwined filaments (Zeldovich et al., 1982; Oort, 1983; Bond et al., 1996).

The difficulties of the neutrino-dominated model became evident in early 1980s. A new scenario was suggested by Blumenthal et al. (1982) and others, where hypothetical particles like axions, gravitinos or photinos play the role of dark matter. Numerical simulations of structure evolution for neutrino and axion-gravitino-photino-dominated universe were made and analysed by Melott et al. (1983). All quantitative characteristics (the connectivity of the structure, the multiplicity of galaxy systems, the correlation function) of this new model fit the observational data well. This model was called the Cold Dark Matter (CDM) model, in contrast to the neutrino-based Hot Dark Matter model. Presently the CDM model with some modifications is the most accepted model of the structure evolution. The properties of the Cold Dark Matter model were analysed in detail by Blumenthal et al. (1984).

The modern cosmological paradigm includes Dark Energy as the basic component of the matter/energy content of the Universe. Direct observational evidence for the presence of Dark Energy comes from distant supernova observations (Perlmutter et al., 1999; Riess et al., 1998) and CMB observations. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite allowed the measurement of the CMB radiation and its power spectrum with a much higher precision (Spergel et al., 2003). The position of the first maximum of the power spectrum depends on the total matter/energy density. Observations confirm the theoretically favoured value 1 in units of the critical cosmological density. Combined CMB, supernova and large-scale distribution data yield for the density of baryonic matter, $\Omega_b = 0.041$, the dark matter density $\Omega_{DM} = \Omega_m - \Omega_b = 0.23$, and the dark energy density $\Omega_\Lambda = 0.73$. These parameters imply that the age of the Universe is 13.7 ± 0.2 gigayears.

Dark energy act as a repulsive force, thus the Universe is presently expanding with an increasing speed. Dark energy also has the effect of freezing the cosmic web. This explains the smoothness of the Hubble flow. The nature of dark matter particles and dark energy is still unknown.

8. CONCLUSIONS

- The discovery of Dark Matter was the result of combined study of galaxies, clusters and their distribution.
- Dark Matter Story is a typical scientific revolution (Tremaine, 1987).
- Evidence for dark matter has been collected independently in many centres.
- There are 2 dark matter problems: dark matter in the Galaxy disk, and dark matter around galaxies and clusters.
- Dark matter in the disk is baryonic (faint stars or Jupiters). The amount is small.
- Dark matter around galaxies is non-baryonic Cold Dark Matter. It constitutes about 0.25 of critical cosmological density.
- Non-baryonic dark matter is needed to start early enough gravitational clustering to form structure. This solves the Big-Bang Nucleosynthesis controversy.

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