Dark Matters, Sept. 24-26, 2012

Lab 1: Dark Matter in Galaxy Clusters

Dynamical Masses, Strong Lensing

Introduction

Although the distribution of galaxies on large scales is quite uniform, on smaller scales, comparable to the mean separation between bright galaxies (about one million parsecs), the galaxy distribution is far from homogeneous. Galaxies are arranged in a variety of large structures from groups of a few galaxies (the nearest example is the Local Group formed by our own Milky Way Galaxy, the Andromeda Galaxy (M31) and a few dozen faint satellite galaxies, including the Magellanic Clouds) and clusters of galaxies containing dozens to hundreds of bright galaxies, to galaxy superclusters and filaments which span tens of millions of parsecs in size and contain many thousands of galaxies. We learn much about the Universe from studying the distribution and properties of these objects.

In the first part of this lab you will estimate the total mass of a galaxy cluster using the virial theorem and compare it to the total mass associated with the stars that populate its galaxies (which can be estimated by measuring the brightness of the galaxies). You will see that the two masses are vastly different, thereby recovering one of the oldest and strongest pieces of observational evidence for the existence of *dark matter* in the universe, which was originally obtained by US astrophysicists Fritz Zwicky and Sinclair Smith in the mid-1930s using very similar method to what you will use.

In the second part of the lab we will explore strong gravitational lensing, and see how lensing gives another method for measuring galaxy and cluster masses.

Picking a Galaxy Cluster

Galaxy clusters are rare objects - only a few percent of all the galaxies in the Universe are in clusters. Much like the sizes of mountains, or the strength of earthquakes, or indeed the masses of individual stars, the distribution of galaxy clusters masses is strongly weighted to lower mass systems - and so the most massive clusters are rarer yet.

These rare massive clusters are quite distinctive though, with astrophysical signatures at a range of wavelengths. To see this, begin by looking at a few known galaxy clusters in World Wide Telescope: set the sky imagery to SDSS (imagery menu bottom left) and find clusters using Search-SIMBAD Search in the menus at the top; search for any of the following objects: Abell 1689, Abell 2219, and the Coma Cluster. This last object is very nearby (on cosmological scales) and is our prototype example of a massive galaxy cluster. Look around the sky nearby the clusters; what do you notice that is different in the cluster regions compared to their surroundings? Have a look at these galaxy clusters at much shorter wavelengths by turning on the ROSAT All Sky Survey layer (RASS - in imagery menu to the bottom left). What do you see at the cluster locations?

Now let's switch to GoogleSky and make some measurements. First, just have a quick look at the galaxy clusters in Places–Cluster_lab_clusters. To move to each object just double click on the name in the places menu. Pick one cluster to work on further.

Now, turn on the `SDSS Galaxy Query` layer - also uncheck the `Photometric Layer` and turn on the `Spectroscopic layer` if not already on. Find galaxies with SDSS spectra (they'll be circled) and look for the spectra of a few of the brightest galaxies in the cluster area. You may need to reclick on your chosen cluster to center the field of view and activate the Query layer at that location. Find an initial approximate redshift of the cluster - do this by looking at the spectrum page (click the symbol, then follow the 'Spectrum' link) of galaxies in the cluster center.

Zoom out to view larger than $15'$ across. Examine the redshifts of galaxies with SDSS at larger distances from the center of the cluster (as marked by the centralised overdensity of bright orangeish galaxies clustered around the brightest galaxy in the cluster). Look for galaxies which which have a similar appearance to the central galaxies. Try to find the most distant galaxy from the center which still has a redshift similar (within perhaps 0.005 in redshift) to that of the cluster. Measure the angular distance between the cluster center and the most distant galaxy using the ruler icon. Note this number — this will be your rough estimate of the cluster extent. Note each galaxy redshift you lok at - you'll use this info below.

Continue to identify more galaxies (the more, the better, but no need to get more than 10-20 or so) within the spatial extent of the cluster with redshifts close to that of the brightest central member galaxies. Record their redshifts and calculate the mean redshift of the cluster as the mean of all these individual redshifts.

Using the angular extent measured above and the estimated redshift of the cluster, compute the actual physical extent of the cluster in Mpc corresponding to the angular extent assuming the Hubble constant value of $H_0 = 70$ km/s/Mpc. To make this calculation simple, use Ned Wright's Javascript cosmology calculator on the web.

Measuring the Cluster Mass using the Virial Theorem

To estimate the total mass of your cluster, you will use the so-called *virial theorem*. This theorem relates the kinetic, K, and potential, W, energies of the system and applies to any self-gravitating system in equilibrium (e.g., it also applies to the Sun and Earth, stellar clusters, etc.). The virial theorem relates the potential (W) and kinetic (K) energy as:

$$W = -2K. \tag{1}$$

The kinetic energy of a cluster of galaxies is

$$K \approx \frac{1}{2} M_{\rm tot} \langle v_{\rm gal}^2 \rangle, \tag{2}$$

where $M_{\rm tot}$ is the *total* gravitating mass of a cluster (which we aim to measure) and $\langle v_{\rm gal}^2 \rangle$ is characteristic velocity disperson of the cluster material. We will estimate the latter by using the dispersion of velocities of the cluster member galaxies about the mean cluster velocity, which can be calculated as

$$\langle v_{\rm gal}^2 \rangle \equiv 3\,\frac{\sum_{i=1}^{N_{\rm gal}} (cz_i - c\bar{z})^2}{N-1} = 3c^2 \times \frac{\sum_{i=1}^{N_{\rm gal}} (z_i)^2 - N_{\rm gal}\bar{z}^2}{N_{\rm gal}-1}. \tag{3}$$

The factor of 3 in front is to account for the fact that redshifts measure only velocity along the line of sight, while the virial theorem applies to velocities along all three dimensions of space; $\bar{z}$ is the mean redshift of the cluster you measured using the $N_{\rm gal}$ galaxies with spectroscopic redshifts z_i, which you have recorded in the previous step.

The potential energy of the self-gravitating system of mass $M_{\rm tot}$ is negative by definition and is given by

$$W = -\alpha \frac{GM_{\rm tot}^2}{R}, \tag{4}$$

where α is a numeric factor of order unity which depends on how mass is distributed within the system, R is the physical extent of the system, and $G = 6.6726 \times 10^{-8}$ cm^3 s^{-2} g^{-1} (in cgs units) is the universal gravitational constant. For the mass distribution expected in clusters $\alpha \approx 0.5$ (good assumption for the accuracy of your measurement which will be to a factor of two or so), and we will use this number to compute the mass.

Combining equations 1, 2, and 4 we get:

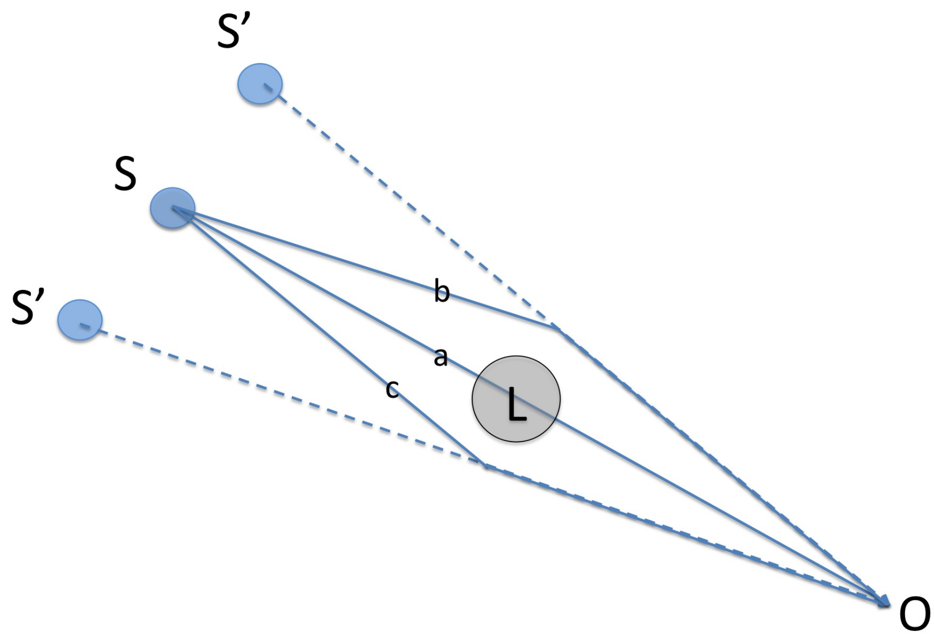
$$M_{\rm tot} = \frac{R\langle v_{\rm gal}^2 \rangle}{\alpha G}. \tag{5}$$

Now, using your tabulated redshifts z_i of galaxy members calculate the mean redshift $\bar{z}$ and $\langle v_{\rm gal}^2 \rangle$ in km/s.

Use equation 5 to estimate the total mass of the cluster within the extent you measured. Take $\alpha = 0.5$. Be careful with units and give the result in solar masses $M_\odot$ (where $M_\odot = 1.989 \times 10^{33}$ g).

Count the number of red bright galaxies around the cluster within R (with or without spectra) to get a rough number of cluster galaxies. Each such bright galaxy has total mass in stars of about $10^{11}\,{\rm M}_\odot$ (give or take a factor of 2-3). This number is determined from the luminosities of these galaxies and the average luminosity and mass of old stars which these galaxies consist of. Estimate the total mass in stars within R using the number of galaxies you counted and the average stellar mass of $10^{11}\,{\rm M}_\odot$. How does this mass compare to the total mass $M_{\rm tot}$ you calculated using the virial theorem? How big is the difference?

The difference between mass in stars and total mass is one of the pieces of observational evidence we have for existence of large amounts of dark gravitating matter in the universe on large scales. Actually, in clusters there are also large amounts of hot plasma mostly consisting of hydrogen and helium nuclei and electrons - as you can see glowing in X-rays and through that plasma's influence on the cosmic microwave background - but the mass of this plasma measured from its X-ray emission still falls far short of explaining the measurements of total cluster masses.



1 Strong Gravitational Lensing

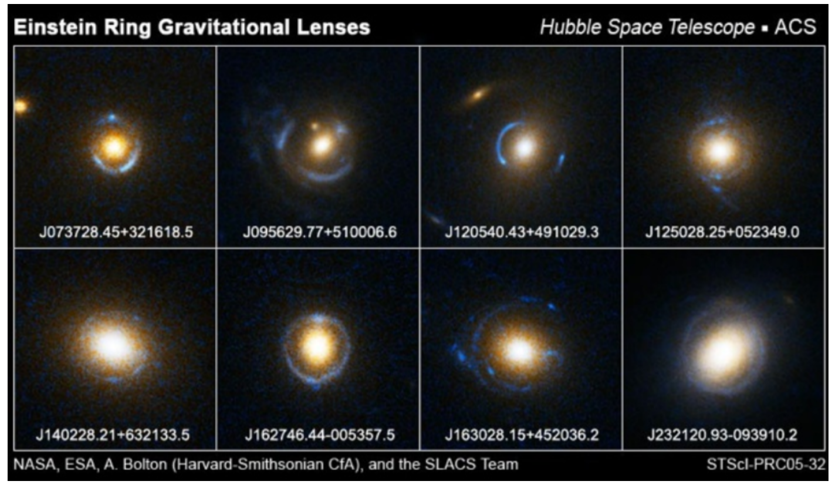
Gravitational lensing - the bending of the paths of photons in the distortion of space-time caused by mass (i.e., by gravity!) - is a prediction of Einstein’s General Theory of Relativity. Gravitational lensing manifests in the universe on a range of mass and length scales. At the smallest scales - the Solar System - the mass of the Sun can cause an subtle shift in the apparent positions of background stars along lines of sight close to the Sun. This effect, only visible during an eclipse (otherwise the light from the Sun precludes observing the background stars) was first observed by Sir Arthur Eddington in 1919, and provided crucial early support to Einstein’s theory, because the observed effect agreed with his predictions.

In the 1930s the astronomer Fritz Zwicky showed that galaxies and clusters of galaxies might be *the* best place in the universe to search for gravitational lensing, and argued that observation of this effect would be the best and cleanest way to measure the mass of these objects.

The basics of gravitational lensing can be understood from a simple diagram, coupled with the knowledge that mass bends the paths of photons[1]. Consider an observer, O, a lensing mass, L, and a source of photons, S. We’ll consider the cosmological scale case here, so S might be a distant galaxy or quasar, and L a group or cluster of galaxies, situated between O and S.

Photons are being emitted by S in all directions, and normally in the absence of a

[1]Formally, photons always travel in straight lines. The effect of mass is to distort space-time such that what locally seems a straight line to a photon represents a curved line when viewed from ’outside’. This is analogous (though in detail not the same) to the experience of driving along straight highway; if asked you would swear that you are driving in a straight line, but in reality you are following a curved path across the curved surface of the Earth.



intervening lens, O will see the source at a position corresponding to the direction from which photons are arriving from the source (case a). With an intervening lens L in place, other photons which normally would not arrive at O have their paths diverted as they pass by L, and end up being observed by O (cases b,c). Under this circumstance the observer 'sees' images of S not at its actual location but at virtual locations corresponding to the direction from which the diverted photons are arriving (images S'). If the alignment between S, L and O is perfectly along a line, and L is circularly symmetric on the sky, then the image S' is a ring on the sky. This is called an Einstein Ring - a sketch of this from the observer's perspective, and actual examples from Hubble Space Telescope observations of lensing systems found in the SDSS are below.

Note that the degree to which the photon's path is deviated is proportional to the mass of L, with a larger mass providing a greater deflection. The apparent radius of the Einstein ring on the sky thus provides a measure of the mass of the lens interior to the ring. The size of the ring is also related to the distance along the line of sight between S, L and O; to robustly determine the mass of the lens we must also know these distances.

Also note that the alignment between S, L and O is rarely perfect, and the lens L is rarely perfectly circular on the sky; the result of this is that complete Einstein rings are very rare and most lensed images S' are arcs which look like portions of a ring image.

We have with us in the lab a conical plastic lens that optically reproduce the effects of gravitational lensing. Use this and the grid of points on the last page of the lab to explore the lensing effect visually; take a few minutes and look at the world around you through this higly distoring optic! Note that only precise alignments yields full rings and that partial arcs are much more typical. Also take a few minutes to explore some images of gravitational lensing recorded by instruments on the Hubble Space Telescope. An extensive gallery of images can be found at `http://hubblesite.org/gallery/album/exotic/gravitational_lens/`

You can also explore gravitational lensing using an app called GravLensHD if you have an ipad or an iphone. If you do have such a device, we'll show you how to work the GravLens app.

The type of lensing we are exploring today is called strong lensing, and is the most obvious manifestation of the bending of photon paths by gravity. Other types of lensing have also been observed.

When the lens involved has the mass of a star (apart from the specific case of the Sun, discussed above) rather than a galaxy or cluster of galaxies, then the radius of the Einstein ring is typically micro-arcseconds. In that case the lensing cannot be observed as a spatial distortion in the directly in the image (no telescope produces images with a resolution fine enough to see micro-arcsecond structures) however the focusing effect of lensing still produces an enhancement in brightness which can be observed. This type of lensing is often referred to as micro-lensing.

A further manifestation of lensing is the weak distortion of the images of background objects which are well away from the observer to lens line-of-sight. In that case the background source is not imaged into a ring or arc, but there is still a discernible pattern of distorted images around the massive object. This pattern can be analyzed statistically, and provides another mass estimate for the lens. This effect, not surprisingly, is know as weak lensing. You can see this effect for yourself; when using the plastic conical optic, note the effect of the lens on the grid of points away from the central peak, as this looks very much like weak lensing in the real universe.

There are now several hundred known strong lensing systems - many of them discovered in sky surveys analysed here at Chicago - and many of them are in the SDSS footprint. In GoogleSky try looking at some of the following locations - use the Location Search tab at top left:
13:43:32.8 +41:55:04
12:09:23.7 +26:40:46
12:26:51.7 +21:52:25

At each of these locations there is a galaxy cluster - somewhat more distant than the clusters in the previous exercise. Each cluster is a strong lens - there is a faint arc-like feature visible in each case. Pick one system, and use the ruler to measure the distance from the arc to the central galaxy of the cluster.

A simple equation provides a good estimate of the mass of the lens if the lens-to-source and observer-to-source distances are known. Typically the lens is about halfway between the observer and source - we will use that approximation here. The initial equation relates the Einstein radius in radians, r$_e$ (the arc-to-cluster-center distance you measured), to the 1D velocity dispersion, σ_v, of the gravitating mass for mass distributions typical of galaxies and clusters:

$$r_e = \frac{4\pi\sigma_v^2}{c^2}\frac{D_{LS}}{D_{OS}} \tag{6}$$

Note that $\frac{D_{LS}}{D_{OS}}$ is approximately 0.5, and that we have from the work above a relationship between velocity dispersion and mass, namely

$$M_{\rm tot} = \frac{R\langle v_{\rm gal}^2\rangle}{\alpha G}. \tag{7}$$

with $\alpha = 0.5$, and noting that $\langle v_{\rm gal}^2\rangle = 3\sigma_v^2$. Thus

$$M_{\rm tot} = \frac{3Rr_ec^2}{2\pi\alpha G}. \tag{8}$$

To solve your equation you'll also need an estimate of the size of the lens. For a cluster of galaxies 1 Mpc (3×10^{22} m) is a decent estimate (R in the above). Putting all this together, what is the mass of your chosen cluster? How does it compare to the masses of clusters computed from the dynamical analysis we did first?